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OPTIMUM SHAPES OF CENTRAL HOLES IN SQUARE PLATES SUBJECTED TO UNIAXIAL UNIFORM LOAD

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Abstract—This paper presents the shapes that will optimize the stress distribution about central holes in square plates subjected to uniform load on two opposite sides of the plate. The study is conducted for a large range of hole to plate widths ratios (*D*/*W*). The stress concentration factor for the optimized holes decreased by as much as 21% when compared to the one associated with a circular hole. Simultaneously, the weight of the plate with optimized hole is reduced by as much as 36% as compared to the circular hole. Coefficient of efficiency of around 0.92 is achieved for all *D*/*W* ratios. The geometry of the optimized holes are presented in a form suitable for use by designers.

INTRODUCTION

This paper is one in a series of papers dealing with the optimization of discontinuities in two dimensional stress fields. The optimum shape of a hole in an infinite plate subjected to uniaxial uniformly distributed load was presented in[1]. The shapes to be given to an optimized central hole in finite plates subjected to uniaxial uniform loading have been given in[2] for different ratios of the diameter of the hole to the width of the plate. The optimized inner boundary shapes of rings with circular outer boundaries subjected to diametral compression have been given in [3] for different ratios of outer to inner diameters. The description of the basic features of the method have been presented in[4, 5]. References to other contributions in the literature can be found in [5] among which the most important one is due to Heywood[6].

OPTIMIZATION PROCEDURE

The method consists in using photoelasticity, in a systematic way, to idealize a configuration so that its boundaries do not have gradients of stress along the length of the boundaries. In other words, the structure will have stresses uniformly, or almost uniformly distributed along the boundary. The procedure permits the direct design of the geometry of the structure rather than the conventional step by step design and analysis, satisfying the requirement that the maximum stress should be lower than an allowable stress and at the same time, the distribution should be as efficient as possible. The geometric constrains for the problem are stripulated initially. A transparent model of the structure is placed in a diffused light circular polariscope. (The material of the model should exhibit birefringence when under load and should be sufficiently sensitive to produce several fringes of interference.) The operator should be able to work on the model with a hand file or portable router at the same time that he looks at the model through the analyzer. The method requires that the operator file away material from the boundaries starting at the points where the stress (and therefore the fringe order) is at a minimum. The filing operation redistributes the fringes. The operator continues filing away material from the low stressed zones of the boundary until as much as possible of the length of the boundary shows the same order of birefringence. This is easy to detect because at that moment, the same fringe falls all along the length of the boundary. If the body has more than one boundary, it may be necessary to operate by steps back and forth from one boundary to the other. In some cases, white light may be more practical than monochromatic light, the objective being then to have the same color along the boundary.

The degree of optimization can be evaluated quantitatively by a coefficient of efficiency:

$$k_{\text{eff}} = \frac{1}{S_2 - S_0} \frac{\int_{S_0}^{S_1} \sigma_t^+ \,\mathrm{d}s}{\sigma_{all}^+} + \frac{\int_{S_1}^{S_2} \sigma_t^- \,\mathrm{d}s}{\sigma_{all}^-}$$

where σ_t is the tangential stress, σ_{all} represents the maximum allowable stress (the positive and negative superscripts referring to tensile and compressive stresses, respectively), S_0 and S_1 are the limiting points of the segment of boundary subjected to tensile stresses and S_1 and S_2 are the limiting points of the segment of boundary with compressive stresses. The significance of the coefficient of efficiency was discussed in Refs. [2, 5]. The above criterion will be used in the present work to evaluate the optimized hole shapes. The design procedure will be particularly useful for components made with brittle materials, or components made with ductile materials subjected to fatigue.

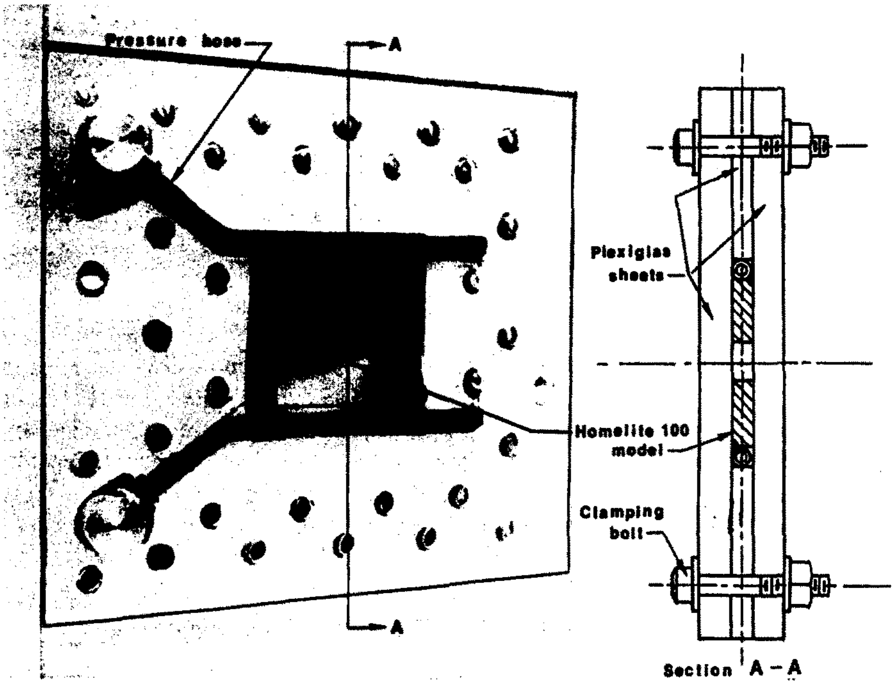


Fig. 1. Loading device used to apply uniform pressure to two opposite sides of a square plate.

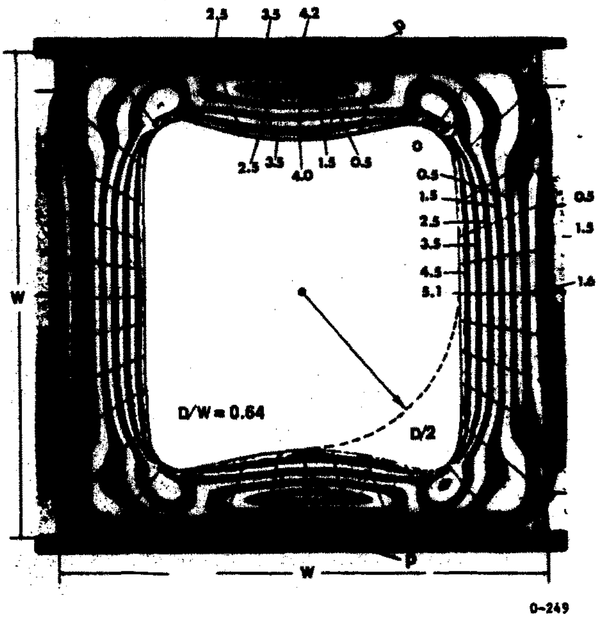


Fig. 2. Typical isochromatic pattern around an optimized hole in a square plate subjected to a uniaxial uniform pressure.

In this paper, using the technique described above, optimized hole shapes are presented in square plates subjected to uniaxial uniform compression.

EXPERIMENTAL DETAILS

Experiments were conducted with 0.23 in. (5.8 mm) thick Homalite-100 plates (fringe constant of 156 lb/in-fr (27.0 kN/m-fr)). The plate size was chosen as 3 × 3 in. (76.2 × 76.2 mm) for all *D*/*W* ratios. Optimization was carried out for *D*/*W* = 0.20; 0.41; 0.56; 0.65; 0.71; 0.79 and 0.89, with the models subjected to uniaxial uniformly distributed compression. Material was removed from the low stress regions by careful hand filing. To improve the precision, in particular at the corner zones, the operator used a binocular magnifier with a set of polarizer and quarter wave plates attached to each of its lenses, during the filing process. The uniform compression on the two parallel boundaries was applied following the methods developed previously [7, 8] and used recently in [9]. The position of a pressurized rubber tube located in the special device is shown in Fig. 1. This loading frame had to be calibrated to determine the

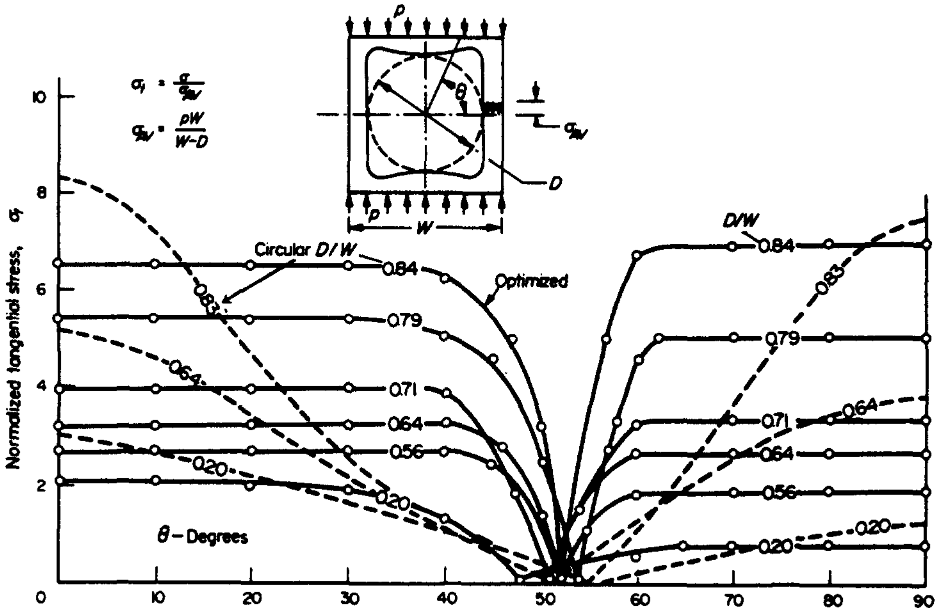


Fig. 3. Stress ratios for points on the boundary of optimized circular holes in a square plate subjected to uniaxial uniform pressure.

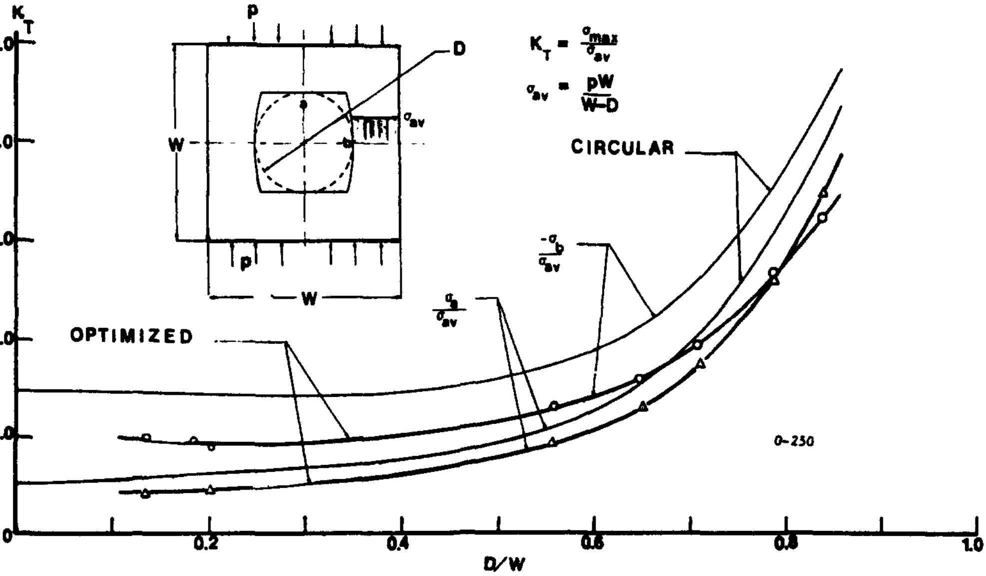


Fig. 4. Comparison of stress concentration factors for an optimized hole vs a circular hole in a square plate subjected to uniaxial uniform pressure.

amount of pressure actually applied to the specimen. For this purpose, a strain gaged load cell was specially designed.

RESULTS

The isochromatic pattern for a typical hole shape is shown in Fig. 2. The stress distributions around circular and optimized holes for the D/W ratios considered are presented in Fig. 3. The s.c.f. for the tensile and compressive regions of the circular and optimized holes for different D/W ratios are plotted in Fig. 4. The information on coefficient of efficiency and percentage weight reduction achieved are given in Fig. 5. The stress distributions on the outer edges are shown in Fig. 6.

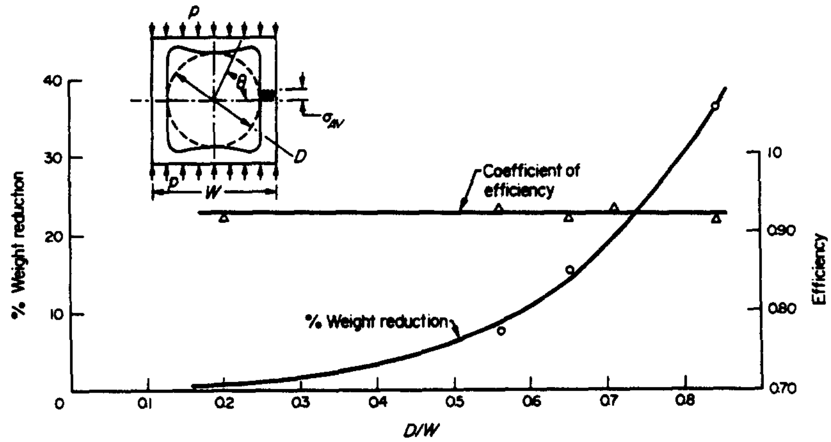


Fig. 5. Percentage weight reduction and coefficient of efficiency for an optimized hole in a square plate subjected to a uniaxial uniform pressure.

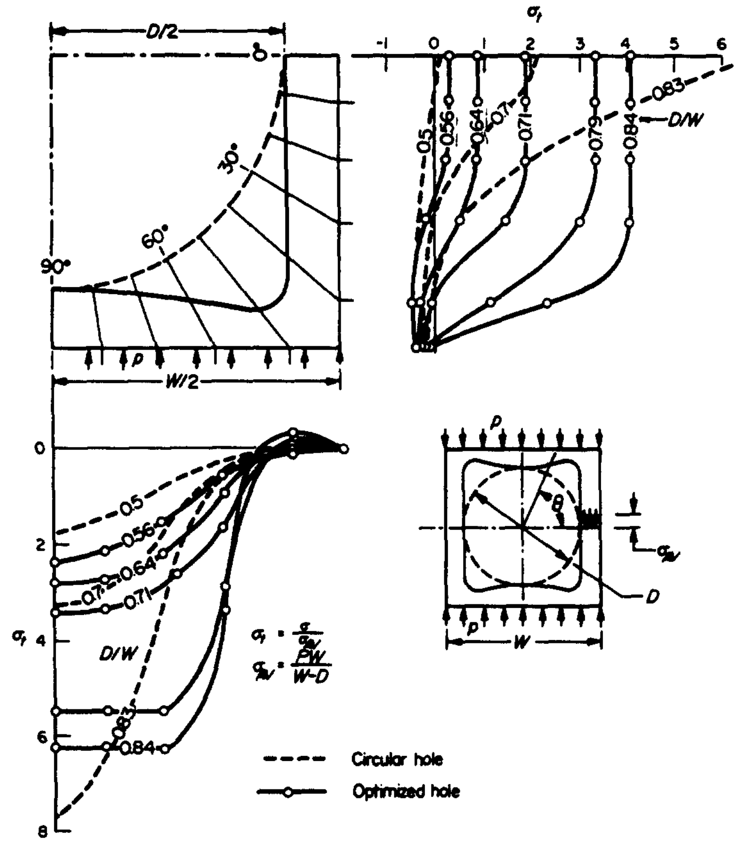
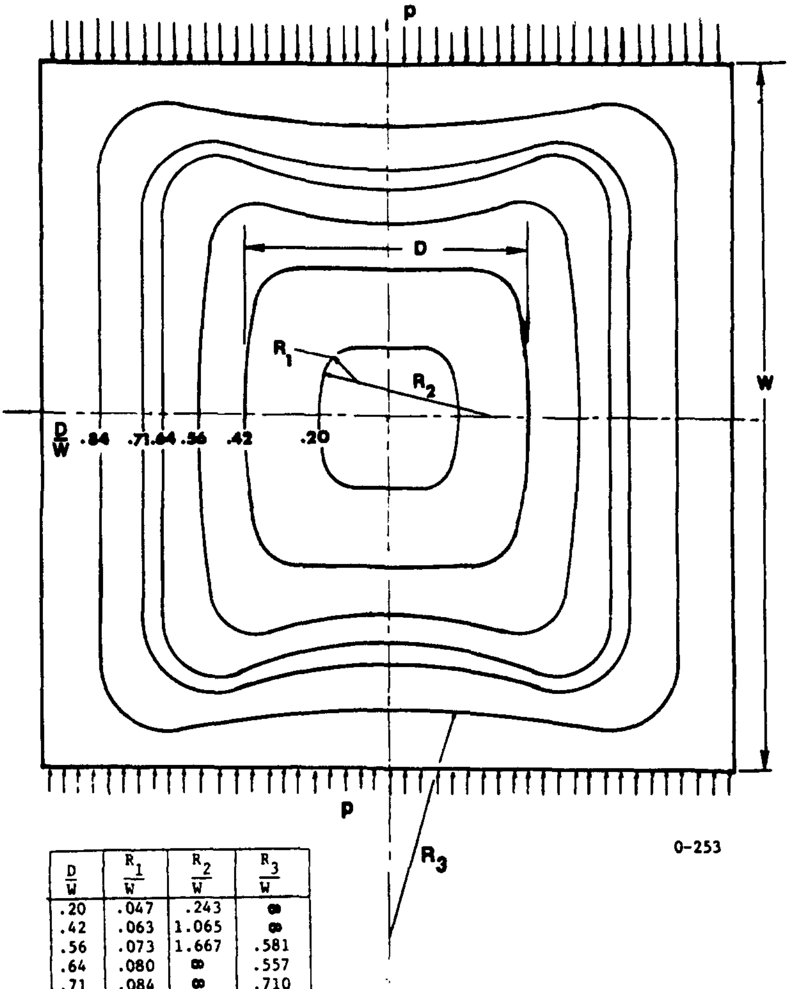


Fig. 6. Stress distribution along the outer boundaries of a square plate, with an optimized hole, subjected to uniform pressure on two opposite sides.



$\frac{D}{W}$	$\frac{R_1}{W}$	$\frac{R_2}{W}$	$\frac{R_3}{W}$
.20	.047	.243	∞
.42	.063	1.065	∞
.56	.073	1.667	.581
.64	.080	∞	.557
.71	.084	∞	.710
.84	.093	∞	1.580

Fig. 7. Optimum shape of a central hole in a square plate subjected to uniform pressure on two opposite sides (as function of D/W).

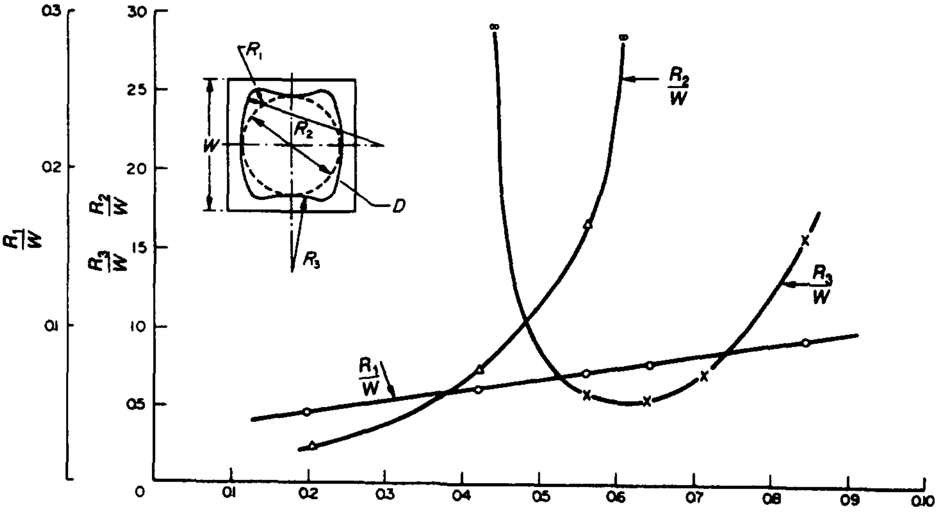


Fig. 8. Radii of the elements of holes producing optimum distribution of stress in square plates subjected to uniaxial uniform pressure.

The experimentally developed optimum hole geometries have been fitted with a combination of circles of different diameters and common tangents at the points of intersection. The hole geometries for the different D/W ratios are given in Fig. 7. The information given in Fig. 7 is consolidated in Fig. 8 in a graphical form to make easy its application.

DISCUSSION

The isochromatic pattern in Fig. 2 shows that the newly developed hole shape exhibits a high degree of optimization with the stresses remaining uniform along major portions of the tensile as well as compressive segments of the boundary. The information in Figs. 3 and 4 shows that, as compared to the circular holes, the optimum shapes have lead to significant reduction in s.c.f., the reduction ranging from 16% for $D/W = 0.14$ to about 21% for $D/W = 0.84$.

A coefficient of efficiency of about 0.92 has been achieved for all D/W ratios, as seen from Fig. 5. The optimum shapes have also lead to significant reduction in weight as compared to the circular holes, in the range of 1% for $D/W = 0.14$ to 36% for $D/W = 0.84$.

Comparison of the optimized shapes and the s.c.f. presented here for square plates with those given in Ref. [2] for long rectangular plates under identical loading conditions (except for the reversal of sign) brings out certain interesting features.

For $D/W < 0.3$, the optimized shapes are found to be identical, whereas for $D/W > 0.3$, the two shapes tend to become quite different especially on the hole edges perpendicular to the load axis. As D/W becomes larger, the hole shapes in square plates are predominantly influenced by the bending effect of the horizontal segments. No such bending effect can be observed in long rectangular plates.

For a very large value of D/W, the problem can be considered as that of a portal frame subjected to uniform load on top. The optimum shape presented can be taken as the shape to be given to a stress optimized portal frame.

While the s.c.f. for square plates decreases initially slightly and then increases monotonically with increase in D/W (Fig. 4), it was found to decrease monotically for long rectangular plates as D/W increases. For a given D/W ratio beyond $D/W = 0.5$, the s.c.f. for square plates is found to be very much higher than that for long rectangular plates. The reason for this increase can be traced to the behavior of the square plate as a portal frame, which increases appreciably the bending stresses in both the vertical and horizontal members.

For large D/W, the vertical elements of the plate become thinner and the fringes are not only parallel to the inner, but also to a large extent, to the outer boundaries indicating a linear variation of stress across the section. It also shows that, for large D/W, as the inner edge gets optimized, the outer edge also tends to become optimum. The distribution of stresses on the outer edges shown in Fig. 6 confirms this. It is also seen from Fig. 6 that the stress distribution becomes favorable also on the loaded edges with the tangential stresses remaining constant over a considerable length.

It is also seen from Fig. 4 that the location of maximum stress shifts from the vertical edge to the horizontal edge for $D/W > 0.81$.

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